Fish, Christopher D.; Jordan, David A.

Connected quantized Weyl algebras and quantum cluster algebras. (English) Zbl 1417.16030

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Summary: For an algebraically closed field $\mathbb{K}$, we investigate a class of noncommutative $\mathbb{K}$-algebras called *connected quantized Weyl algebras*. Such an algebra has a PBW basis for a set of generators $\{x_1,\dots,x_n\}$ such that each pair satisfies a relation of the form $x_ix_j=q_{ij}x_jx_i+r_{ij}$, where $q_{ij}\in\mathbb{K}^*$ and $r_{ij}\in\mathbb{K}$, with, in some sense, sufficiently many pairs for which $r_{ij}\neq 0$. For such an algebra it turns out that there is a single parameter q such that each $q_{ij}=q^{\pm 1}$. Assuming that $q\neq\pm 1$, we classify connected quantized Weyl algebras, showing that there are two types *linear* and *cyclic*. When q is not a root of unity we determine the prime spectra for each type. The linear case is the easier, although the result depends on the parity of n, and all prime ideals are completely prime. In the cyclic case, which can only occur if n is odd, there are prime ideals for which the factors have arbitrarily large Goldie rank. We apply connected quantized Weyl algebras to obtain presentations of two classes of quantum cluster algebras. Let $n\geq 3$ be an odd integer. We present the quantum cluster algebra of a Dynkin quiver of type A_{n-1} as a factor of a linear connected quantized Weyl algebra by an ideal generated by a central element. We also consider the quiver $P^{(1)}_{n+1}$ identified by Fordy and Marsh in their analysis of periodic quiver mutation. When n is odd, we show that the quantum cluster algebra of this quiver is generated by a cyclic connected quantized Weyl algebra in n variables and one further generator. We also present it as the factor of an iterated skew polynomial algebra in $n+2$ variables by an ideal generated by a central element. For both classes, the quantum cluster algebras are simple noetherian.

We establish Poisson analogues of the results on prime ideals and quantum cluster algebras. We determine the Poisson prime spectra for the semiclassical limits of the linear and cyclic connected quantized Weyl algebras and show that, when n is odd, the cluster algebras of A_{n-1} and $P^{(1)}_{n+1}$ are simple Poisson algebras that can each be presented as a Poisson factor of a polynomial algebra, with an appropriate Poisson bracket, by a principal ideal generated by a Poisson central element.

MSC:

16S36 Ordinary and skew polynomial rings and semigroup rings
13F60 Cluster algebras
16D30 Infinite-dimensional simple rings (except as in 16Kxx)
16N60 Prime and semiprime associative rings
16W20 Automorphisms and endomorphisms
17B63 Poisson algebras

Keywords:

Weyl algebras; quantum cluster algebras

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