Automorphisms of the Lie algebra of vector fields on affine $n$-space. (English) Zbl 1418.17051


Summary: We study the vector fields $\text{Vec}(\mathbb{A}^n)$ on affine $n$-space $\mathbb{A}^n$, the subspace $\text{Vec}^c(\mathbb{A}^n)$ of vector fields with constant divergence, and the subspace $\text{Vec}^0(\mathbb{A}^n)$ of vector fields with divergence zero, and we show that their automorphisms, as Lie algebras, are induced by the automorphisms of $\mathbb{A}^n$:

$$\text{Aut}(\mathbb{A}^n) \rightarrow \text{Aut}_{\text{Lie}}(\text{Vec}(\mathbb{A}^n)) \rightarrow \text{Aut}_{\text{Lie}}(\text{Vec}^0(\mathbb{A}^n)) \rightarrow \text{Aut}_{\text{Lie}}(\text{Vec}^0(\mathbb{A}^n)).$$

This generalizes results of the second author obtained in dimension 2, see the second author, Lie subalgebras of vector fields and the Jacobian conjecture, arxiv:1311.0232 (2013)]. The case of $\text{Vec}(\mathbb{A}^n)$ goes back to V. S. Kulikov [Russ. Acad. Sci., Izv., Math. 41, No. 2, 351–365 (1993); translation from Izv. Ross. Akad. Nauk, Ser. Mat. 56, No. 5, 1086–1103 (1992; Zbl 0796.14008)]. This generalization is crucial in the context of infinite-dimensional algebraic groups, because $\text{Vec}^c(\mathbb{A}^n)$ is canonically isomorphic to the Lie algebra of $\text{Aut}(\mathbb{A}^n)$, and $\text{Vec}^0(\mathbb{A}^n)$ is isomorphic to the Lie algebra of the closed subgroup $\text{SAut}(\mathbb{A}^n) \subset \text{Aut}(\mathbb{A}^n)$ of automorphisms with Jacobian determinant equal to 1.

MSC:

17B66 Lie algebras of vector fields and related (super) algebras
14R15 Jacobian problem

Keywords:
automorphisms; vector fields; Lie algebras; affine $n$-space

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References:


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