In this work the authors study how geometrical finiteness properties of spaces and groups can be deduced from finiteness properties of their algebraic models.

For specifying this relation, recall that a commutative differential graded algebra, \( A^* \), is a \( q \)-model of a space \( X \) if it has the same homotopy \( q \)-type as the Sullivan piecewise polynomial forms on \( X \) [D. Sullivan, Publ. Math., Inst. Hautes Étud. Sci. 47, 269–331 (1977; Zbl 0374.57002)]. Also, \( A \) is said \( q \)-finite if \( A^0 = \mathbb{Q} \) and the vector space \( \bigoplus_{i \leq q} A^i \) is finite dimensional. Finally, a path-connected space \( X \) is \( q \)-finite if it is homotopy equivalent to a CW-complex with finite \( q \)-skeleton.

The authors address the question: “When does a \( q \)-finite space admit a \( q \)-finite \( q \)-model?” They obtain a complete answer for \( q = 1 \): a space \( X \) with finitely generated fundamental group \( \pi \) admits a 1-finite 1-model if, and only if, the Malcev Lie algebra of \( \pi \) is the lower central series completion of a finitely presented Lie algebra. In the general case, an infinitesimal obstruction to a positive answer is given.

Reviewer: Daniel Tanré (Villeneuve d’Ascq)

MSC:

55P62 Rational homotopy theory
17B01 Identities, free Lie (super)algebras
20F14 Derived series, central series, and generalizations for groups
20J05 Homological methods in group theory
55N25 Homology with local coefficients, equivariant cohomology

Keywords:

Malcev Lie algebra; completion; lower central series

Full Text: DOI

References:
