Hamilton, M. J. D.
Bi-Lagrangian structures on nilmanifolds. (English) Zbl 1422.53024

By definition a bi-Lagrangian structure on a smooth manifold consists of a symplectic form together with a pair of supplementary Lagrangian foliations. In the literature, bi-Lagrangian structures are also known as Künneth or para-Kähler structures.

The author classifies those nilpotent Lie algebras in dimensions 2, 4, and 6 which admit a bi-Lagrangian structure. His findings can be summarized as follows:

1. In dimension 2 there is a single nilpotent Lie algebra up to isomorphism, and it admits a bi-Lagrangian structure.
2. In dimension 4 there are three Lie algebras up to isomorphism, each of which admits a symplectic form. Of these, two admit a bi-Lagrangian structure and one does not.
3. In dimension 6 there are 26 nilpotent Lie algebras up to isomorphism that admit a symplectic form. Of these, 16 admit a bi-Lagrangian structure and 10 do not.

For each of these examples of bi-Lagrangian structures on a nilpotent Lie algebra, the author calculates the curvature of the canonical connection associated to the bi-Lagrangian structure. His calculations show that:

1. The canonical connection is flat for all examples in dimension 2 and 4.
2. In dimension 6, of the 16 examples admitting a bi-Lagrangian structure, the canonical connection is flat for 8 of them and non-flat for the remaining 8, and all 16 are Ricci-flat.

Reviewer: James Hebda (St. Louis)

MSC:
53C15 General geometric structures on manifolds (almost complex, almost product structures, etc.)
53D12 Lagrangian submanifolds; Maslov index
53C12 Foliations (differential geometric aspects)
57R30 Foliations in differential topology; geometric theory
22E25 Nilpotent and solvable Lie groups
53D05 Symplectic manifolds (general theory)

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