Let $S$ be a connected, compact, orientable surface with a finite non-empty set of marked points. A polygonalisation of $S$ is a multi-arc on $S$ connecting marked points which decomposes $S$ into polygons. The polygonalisation complex $\mathcal{P}(S)$ of $S$ is a cube complex (has a decomposition into Euclidean cubes glued along their faces by isometries) whose vertices are polygonalisations (two vertices are connected by an edge if they differ by a single arc). The polygonalisation complex $\mathcal{P}(S)$ contains the flip graph $\mathcal{F}(S)$ as a subcomplex whose vertices are triangulations of $S$ (two triangulations are connected by an edge if they differ by a flip, replacing a diagonal of a quadrilateral of a triangulation by the other diagonal). The mapping class group $\text{Mod}(S)$ acts on both $\mathcal{P}(S)$ and $\mathcal{F}(S)$; since, in general, $\text{Mod}(S)$ is not a CAT(0) group \[ M. \text{ Kapovich and B. Leeb, Math. Ann. 306, No. 2, 341–352 (1996; Zbl 0856.20024)} \], $\mathcal{P}(S)$ is not a CAT(0) cube complex (a cube complex is non-positively curved if it satisfies Gromov’s link condition, and it is CAT(0) if it is non-positively curved and simply connected).

The authors characterize the vertices where Gromov’s link condition fails and use this for proving that, generically, $\text{Mod}(S)$ is the automorphism group of $\mathcal{P}(S)$ (it is known to be the automorphism group of the flip graph $\mathcal{F}(S)$). They show that $\mathcal{P}(S)$ has many properties of a CAT(0) cube complex, and in particular a rich hyperplane structure closely related to the arc graph $\mathcal{A}(S)$ (again with automorphism group $\text{Mod}(S)$; arcs are connected by an edge here if they are disjoint). The authors show that there is a natural one-to-one correspondence between the hyperplanes of $\mathcal{P}(S)$ and the arcs on $S$. Applying rigidity results for arc graphs, they also show that, generically, different surfaces have different polygonalisation complexes.

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References:


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