The average case complexity of the parallel prefix problem. (English) Zbl 1422.68117

Summary: We analyse the average case complexity of evaluating all prefixes of an input vector over a given semigroup. As computational model circuits over the semigroup are used and a complexity measure for the average delay of such circuits, called time, is introduced. Based on this notion, we then define the average case complexity of a computational problem for arbitrary input distributions.

For highly nonuniform distributions the average case complexity turns out to be as large as the worst case complexity. Thus, in order to make the average case analysis meaningful we also develop a complexity measure for distributions.

Using this framework we show that two $n$-bit numbers can be added with an average delay of order $\log \log n$ for a large class of distributions. We then give a complete characterization of the average case complexity of the parallel prefix problem with respect to the underlying semigroup. By considering a related reachability problem for finite automata it is shown that the complexity only depends on a property of the semigroup we will call a confluence.

Our analysis yields that only two different cases can arise for the reachability question. We show that the parallel prefix problem either can be solved with an average delay of order $\log \log n$, that means with an exponential speedup compared to the worst case, or in case of nonconfluent semigroups that no speedup is possible. Circuit designs are presented that for confluent semigroups achieve the optimal double logarithmic delay while keeping the circuit size linear.

The analysis and results are illustrated at some concrete functions. For the $n$-ary Boolean OR, THRESHOLD and PARITY, for example, the average case circuit delay is determined exactly up to small constant factors for arbitrary distributions.

Finally, we determine the complexity of the reachability problem itself and show that it is at most quadratic in the size of the semigroup.

For the entire collection see Zbl 0844.00024.

MSC:

68Q25 Analysis of algorithms and problem complexity
20M35 Semigroups in automata theory, linguistics, etc.
68Q45 Formal languages and automata
94C10 Switching theory, application of Boolean algebra; Boolean functions (MSC2010)

Keywords:

average case; average delay; finite automaton; reachability problem; input gate

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References:


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