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Integral points on conic log K3 surfaces. (English) Zbl 1423.11119
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This paper gives explicit sufficient conditions for the existence of integral points on certain schemes which are fibered into affine conics. Examples of such schemes include the case where the scheme is geometrically a smooth log K3 surfaces. A prototypical result is given in the following theorem.

Theorem: Let S be a finite set of places of \mathbb{Q} containing $2, \infty$. For each $i = 1, \dots, 4$, let $c_i, d_i \in \mathbb{Z}_S$ be a pair of S -coprime S -integers such that $\Delta_{i,j} := c_j d_i - c_i d_j$ is non-zero for $i \neq j \in \{1, \dots, 4\}$. Let $\mathcal{Y} \rightarrow \mathbb{P}_S^1$ be the pencil of affine conics determined inside $\mathcal{O}(-1) \oplus \mathcal{O}(-1)$ by an equation of the form

$$(c_1 t + d_1 s)(c_2 t + d_2 s)x^2 + (c_3 t + d_3 s)(c_4 t + d_4 s)y^2 = 1.$$

Assume that the classes $[-1], [\Delta_{1,2}], [\Delta_{1,3}], [\Delta_{1,4}], [\Delta_{2,3}], [\Delta_{2,4}], [\Delta_{3,4}]$ are distinct linearly independent classes in $\mathbb{Q}^*/(\mathbb{Q}^*)^2$. Then \mathcal{Y} has an S -integral point.

Examples of \mathcal{Y} include log K3 surfaces. The method used in this paper is the integral descent-fibration method, which is an extension of the fibration-method originally developed by Swinnerton-Dyer.

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MSC:

- [11G99](#) Arithmetic algebraic geometry (Diophantine geometry)
- [14G99](#) Arithmetic problems in algebraic geometry; Diophantine geometry

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