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Homogenized conductivity tensor and absorption function of a locally periodic porous medium. (Russian. English summary) Zbl 1424.35123

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Summary: We study a process of stationary diffusion in locally-periodic porous media with nonlinear absorption at the pore boundary. This process is described by a boundary-value problem for an elliptic equation considered in a complex perforated domain, with a nonlinear third boundary condition on the perforation boundary. In view of the smallness of the local scale of porosity of the media and the complexity of the perforated domain, the direct solution of such boundary-value problems is almost impossible. Therefore, a natural approach in this situation is to study the asymptotic behavior of the solution when the microstructure scale tends to 0, and the transition to the homogenized macroscopic model of the process. Our earlier papers were devoted to homogenization of the diffusion equation in a wide class of non-periodically perforated domains: strongly-connected domains, which includes locally-periodically perforated domains. In these works, a homogenized model is proposed, the coefficients of which are expressed in terms of "mesoscopic" (local energy) characteristics of the media, which are determined in small cubes, the size of which, however, are much larger than the microstructure scale. The convergence theorems are proved under the conditions of the existence of limiting densities of "mesoscopic" characteristics, the fulfillment of which is generally difficult to show, but in a number of specific situations this can be done. In this paper, we show the fulfillment of these conditions and, by studing them, we obtain explicit formulas for the effective characteristics of the locally-periodic porous medium: a conductivity tensor and a function of absorption.

MSC:

35G60 Boundary value problems for systems of nonlinear higher-order PDEs
35Q79 PDEs in connection with classical thermodynamics and heat transfer

Keywords:

homogenization; stationary diffusion; non-linear third boundary value problem; locally periodic porous medium

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