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Goldman-Turaev formality from the Knizhnik-Zamolodchikov connection. (Connexion de Knizhnik-Zamolodchikov et formalité pour la bigèbre de Lie de Goldman-Turaev.) (English. French summary) [Zbl 1425.17029]

Summary: For an oriented 2-dimensional manifold $\Sigma$ of genus $g$ with $n$ boundary components, the space $C\pi_1(\Sigma)/[C\pi_1(\Sigma), C\pi_1(\Sigma)]$ carries the Goldman-Turaev Lie bialgebra structure defined in terms of intersections and self-intersections of curves. Its associated graded Lie bialgebra (under the natural filtration) is described by cyclic words in $H_1(\Sigma)$ and carries the structure of a necklace Schedler Lie bialgebra. The isomorphism between these two structures in genus zero has been established in [G. Massuyeau, Quantum Topol. 9, No. 1, 39–117 (2018; Zbl 1393.57016)] using Kontsevich integrals and in [A. Alekseev et al., Adv. Math. 326, 1–53 (2018; Zbl 1422.57053)] using solutions of the Kashiwara-Vergne problem.

In this note, we give an elementary proof of this isomorphism over $\mathbb{C}$. It uses the Knizhnik-Zamolodchikov connection on $\mathbb{C}\{z_1, \ldots, z_n\}$. We show that the isomorphism naturally depends on the complex structure on the surface. The proof of the isomorphism for Lie brackets is a version of the classical result by N. Hitchin [NATO ASI Ser., Ser. C, Math. Phys. Sci. 488, 69–112 (1997; Zbl 0867.53027)]. Surprisingly, it turns out that a similar proof applies to cobrackets.

Furthermore, we show that the formality isomorphism constructed in this note coincides with the one defined in [Alekseev (loc. cit.)] if one uses the solution of the Kashiwara-Vergne problem corresponding to the Knizhnik-Zamolodchikov associator.

MSC:
17B62 Lie bialgebras; Lie coalgebras
17B70 Graded Lie (super)algebras

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References:
    English translation: · Zbl 0422.57005


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