This is an extremely nice article. The authors study on a compact metric-measure space \((X, d, \mu)\) of finite measure the \(\rho\)-Laplacian \(\Delta^\rho_X u : L^2(X) \to L^2(X)\) for \(\rho > 0\) defined by

\[
\Delta^\rho_X u(x) = \frac{1}{\rho^2 \mu(B_\rho(x))} \int_{B_\rho(x)} (u(x) - u(y)) d\mu(y).
\]

This is a nice candidate for a Laplacian on this general setting because of the following facts: (i) if \(X\) is a Riemannian space of dimension \(n\), then \(\Delta^\rho_X\) converges as \(\rho \to 0\) to the Laplace–Beltrami operator associated to \(X\) multiplied by \(-\frac{1}{2(n+2)}\); (ii) if \(X\) is a discrete space, then \(\Delta^\rho_X\) is just the weighted graph Laplacian.

The main goal of the article is to study, for a fixed \(\rho > 0\), spectral properties of \(\Delta^\rho_X\). It is shown that if \((X_n, d_n, \mu_n)\) is a sequence converging to \((X, d, \mu)\) in the sense of \([K. Fukaya; Invent. Math. 87, 517–547 (1987; Zbl 0589.58034)]\) satisfying that \(d\) is a length metric and there is \(\Lambda > 0\) such that \(\frac{\mu(B_{r_1}(x))}{\mu(B_{r_2}(x))} \leq (\frac{r_1}{r_2})^\Lambda\) for all \(x \in X\) and \(r_1 \geq r_2 > 0\), then

\[
\lambda_k(X, \rho) = \lim_{n \to \infty} \lambda_k(X_n, \rho)
\]

for all \(k\) such that \(\lambda_k(X, \rho) < \rho^{-2}\). Here, \(\lambda_k(X, \rho)\) and \(\lambda_k(X_n, \rho)\) denote the \(k\)-th eigenvalue of \(\Delta^\rho_X\) and \(\Delta^\rho_{X_n}\) respectively.

The paper also contains a very instructive section of examples, gives a Weyl-type estimate, and provides a pleasant introductory section.

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MSC:
58J50 Spectral problems; spectral geometry; scattering theory on manifolds
58J60 Relations of PDEs with special manifold structures (Riemannian, Finsler, etc.)
53C23 Global geometric and topological methods (à la Gromov); differential geometric analysis on metric spaces

Keywords:
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