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On asymptotic global error estimation and control of finite difference solutions for semilinear parabolic equations. (English) Zbl 1425.65087

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Summary: The aim of this paper is to extend the global error estimation and control addressed in [*J. Lang* and *J. G. Verwer*, SIAM J. Sci. Comput. 29, No. 4, 1460–1475 (2007; Zbl 1145.65047)] for initial value problems to finite difference solutions of semilinear parabolic partial differential equations. The approach presented there is combined with an estimation of the PDE spatial truncation error by Richardson extrapolation to estimate the overall error in the computed solution. Approximations of the error transport equations for spatial and temporal global errors are derived by using asymptotic estimates that neglect higher order error terms for sufficiently small step sizes in space and time. Asymptotic control in a discrete L_2-norm is achieved through tolerance proportionality and uniform or adaptive mesh refinement. Numerical examples are used to illustrate the reliability of the estimation and control strategies.

MSC:

65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs
35K20 Initial-boundary value problems for second-order parabolic equations
35K58 Semilinear parabolic equations
65M15 Error bounds for initial value and initial-boundary value problems involving PDEs
65M50 Mesh generation, refinement, and adaptive methods for the numerical solution of initial value and initial-boundary value problems involving PDEs

Keywords:

numerical integration for PDEs; method of lines; finite difference method; asymptotic global error estimation; asymptotic global error control; defects and local errors

Software:

ROS3P

Full Text: DOI

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