Lin, Francesco
PIN(2)-monopole Floer homology and the Rokhlin invariant. (English) Zbl 1427.57026

Earlier, the author of the present paper introduced several flavors of $\text{Pin}(2)$-equivariant Floer homology for closed oriented three-manifolds $Y$, equipped with a self-conjugate spin-c structure. This paper gives a calculation of the bar flavor of the $\text{Pin}(2)$-equivariant monopole Floer homology.

Recall that the bar flavor $\mathcal{HS}(Y, s)$ (and its close relative $\mathcal{HM}(Y, s)$) reflects the Seiberg-Witten equations only in a neighborhood of the reducible solutions, where the reducibles are given as a copy of the Picard torus of the underlying three-manifold $Y$. It is thus the simplest of the equivariant Floer homology packages associated to $(Y, s)$. The bar flavor of ordinary monopole Floer homology $\mathcal{HM}(Y, s)$ was studied by P. Kronheimer and T. Mrowka [Monopoles and three-manifolds. Cambridge: Cambridge University Press (2007; Zbl 1158.57002)], where they showed that it is determined by the index of the family of Dirac operators over the Picard torus (recovering the actual homology from this description is not itself obvious).

Morally, $\mathcal{HS}(Y, s)$ is some kind of equivariant refinement of $\mathcal{HM}(Y, s)$, and the paper under review shows that this invariant is determined by the equivariant (with respect to the action of conjugation) index of the natural family of Dirac operators over the index bundle.

For practical purposes, Lin then shows that the equivariant family index of the family of Dirac operators associated to $(Y, s)$ is determined by the Rokhlin invariants of the collection of spin structures inducing the spin-c structure $s$, along with the triple cup product (namely, $(Y_1, s_1)$ and $(Y_2, s_2)$ will have the same $\mathcal{HS}(Y_i, s_i)$ if there is a bijection of the spin structures, compatible with an identification of the triple cup products).

In a closing section, the author provides some explicit calculations. Indeed, the methods of the paper seem to make the calculation of $\mathcal{HS}(Y, s)$ practicable (or at least partially so) in many situations of interest. This paper is a nice further demonstration of the richness of the $\text{Pin}(2)$-equivariant theory $\mathcal{HS}(Y, s)$.

Reviewer: Matthew Stoffregen (Cambridge)

MSC:

57R58 Floer homology
57K30 General topology of 3-manifolds
57K18 Homology theories in knot theory (Khovanov, Heegaard-Floer, etc.)
58J20 Index theory and related fixed-point theorems on manifolds

Keywords:

monopole Floer homology; $\text{Pin}(2)$-equivariant Floer homology; Rokhlin invariant

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References:
