Upsilon-like concordance invariants from $\mathfrak{s}_n$ knot cohomology. (English) Zbl 1428.57008


The authors define new knot concordance invariants from (specific deformations of) Khovanov-Rozansky $\mathfrak{s}_n$-homologies. This invariant, called $\Upsilon_n$, is analogous to the Ozsváth-Stipsicz-Szabó $\Upsilon$ invariant from [P. S. Ozsváth et al., Adv. Math. 315, 366–426 (2017; Zbl 1383.57020)]. Furthermore, they provide examples where $\Upsilon_n$ can be used to obstruct sliceness and concordance to alternating links, whereas $\Upsilon$ and other known invariants fail. Finally, the two authors define a new concordance invariant $S$, which is a direct summand of the equivariant Khovanov-Rozansky complex, called the equivariant Rasmussen invariant. This invariant 'dominates' a number of known concordance invariants arising from Khovanov-Rozansky homologies.

To sketch the construction of $\Upsilon_n$ we start by reviewing the construction of $\Upsilon$. In knot Floer homology one can 'blend' the algebraic and the Alexander filtrations on $\text{CFK}^-(K)$ into a unique filtration which depends on a parameter $t \in [0, 2]$. The least possible filtered degree (in homology) where one can find non-trivial elements gives the value of $\Upsilon_K(t)$. The construction of the invariant $\Upsilon_n$ proceeds in a similar, albeit different, way. Once a potential (i.e. a monic polynomial of degree $n$) is fixed one can define a deformation of Khovanov-Rozansky $\mathfrak{s}_n$-homology with respect to the chosen potential. In this case we pick the potential to be $x^n - x^{n-1}$, then there is a well defined (diagram dependent) chain $\psi_D$. Similarly to the case of knot Floer homology, one 'blends' the quantum and $x$-filtration on the deformed complex into a unique, $t$-dependent, filtration. The value of $\Upsilon_n(K)(t)$ is the least possible filtered degree where the homology class of $[\psi_D]$ does not vanish.

Similarly to $\Upsilon$, the invariant $\Upsilon_n$ is a piecewise linear function, and provides a lower bound on the slice genus. However, differently from $\Upsilon$, the function $\Upsilon_n$ in not a concordance homomorphism (but a quasi-homomorphism), and provides highly non trivial information for quasi-alternating knots.

Finally, towards the end of the paper the authors prove that (i) the stable (i.e. up to acyclic summands) homotopy type of the equivariant Khovanov-Rozansky homology is a link invariant, and (ii) the existence of the equivariant Rasmussen invariant $S$. Fact (i) is a weaker analogue of the concordance invariance of the knot Floer chain complex up to stable homotopy, due to J. Hom [J. Knot Theory Ramifications 26, No. 2, Article ID 1740015, 24 p. (2017; Zbl 1360.57002)]. While fact (ii) is reminescent (and can be seen as a partial extension of) the work of J. Pardon concerning the Khovanov-Lee homology of links [Algebr. Geom. Topol. 12, No. 2, 1081–1098 (2012; Zbl 1263.57007)].

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References:
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