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On generalized Erdős-Ginzburg-Ziv constants of $C_r^n$. (English) Zbl 1429.11053


Summary: Let $G$ be an additive finite abelian group with exponent $\exp(G) = n$. For any positive integer $k$, the $k$th Erdős-Ginzburg-Ziv constant $s_{kn}(G)$ is defined as the smallest positive integer $t$ such that every sequence $S$ in $G$ of length at least $t$ has a zero-sum subsequence of length $kn$. It is easy to see that $s_{kn}(C_r^n) \geq (k + r)n - r$ where $n, r \in \mathbb{N}$. S. Kubertin [Acta Arith. 116, No. 2, 145–152 (2005; Zbl 1076.11011)] conjectured that the equality holds for any $k \geq r$. In this paper, we prove the following results:

(1) For every positive integer $k \geq 6$, we have

$$s_{kn}(C_3^n) = (k + 3)n + O \left( \frac{n}{\ln n} \right).$$

(2) For every positive integer $k \geq 18$, we have

$$s_{kn}(C_4^n) = (k + 4)n + O \left( \frac{n}{\ln n} \right).$$

(3) For $n \in \mathbb{N}$, assume that the largest prime power divisor of $n$ is $p^a$ for some $a \in \mathbb{N}$. For any fixed $r \geq 5$, if $p^t \geq r$ for some $t \in \mathbb{N}$, then for any $k \in \mathbb{N}$ we have

$$s_{kp^tn}(C_r^n) \leq (kp^t + r)n + c_r \frac{n}{\ln n},$$

where $c_r$ is a constant that depends on $r$.

Our results verify the conjecture of Kubertin asymptotically in the above cases.

MSC:

11B75 Other combinatorial number theory
11B50 Sequences (mod $m$)

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zero-sum theory; generalized Erdős-Ginzburg-Ziv constants

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References:
