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Finite field restriction estimates based on Kakeya maximal operator estimates. (English)
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J. Eur. Math. Soc. (JEMS) 21, No. 12, 3649-3707 (2019).

Summary: In the finite field setting, we show that the restriction conjecture associated to any one of a large family of $d = 2n+1$ dimensional quadratic surfaces implies the $n+1$-dimensional Kakeya conjecture (Dvir's theorem). This includes the case of the paraboloid over finite fields in which -1 is a square. We are able to partially reverse this implication using the sharp Kakeya maximal operator estimates of *J. S. Ellenberg* et al. [Mathematika 56, No. 1, 1–25 (2010; Zbl 1189.42010)] to establish the first finite field restriction estimates beyond the Stein-Tomas exponent in this setting.

MSC:

42B10	Fourier and Fourier-Stieltjes transforms and other transforms of Fourier type	Cited in **5** Documents
42B25	Maximal functions, Littlewood-Paley theory	
14G15	Finite ground fields in algebraic geometry	
52C17	Packing and covering in n dimensions (aspects of discrete geometry)	
11T24	Other character sums and Gauss sums	

Keywords:

Fourier transform; restriction conjecture; Kakeya conjecture; finite fields

Full Text: DOI arXiv

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