We define each pair $\varepsilon$ algebra.

Given a multiplicatively antisymmetric $n \times n$ matrix $q$ over an algebraically closed field $k$, we can construct the $q$-commutative power series ring $R = k_q[[x_1, \ldots, x_n]]$ and Laurent series ring $L = k_q[[x_1^{\pm 1}, \ldots, x_n^{\pm 1}]]$, with multiplication given by $x_ix_j = q_{ij}x_jx_i$. In the present study, each $q_{ij}$ is assumed to be a root of unity, in which case it follows that $R$ is finite over its center, and so must be a polynomial identity (PI) algebra.

Let $\varepsilon$ be a generator of the multiplicative cyclic group $\langle q_{ij} \rangle$, and define $\ell$ to be the order this group. For each pair $i, j$ of indices such that $1 \leq i, j \leq n$, we select $h_{ij}$ such that $q_{ij} = \varepsilon^{h_{ij}}$. Using the matrix $(h_{ij})$, we can define a homomorphism $H : \mathbb{Z}^n \to (\mathbb{Z}/\ell\mathbb{Z})^n$. We then define $h$ to be the cardinality of the image of $H$. The authors demonstrate that $H$ has PI degree $\sqrt{h}$ and $L$ is an Azumaya algebra of $PI$ degree $\sqrt{h}$.

We define $\sigma : \mathbb{Z}^n \times \mathbb{Z}^n \to k^\times$ to be the alternating bicharacter given by $\sigma(s, t) = \prod_{i=1}^n q_{ij}^{s_i t_j}$. Using this, we can then define a free abelian subgroup of $\mathbb{Z}^n$ by $S = \{s \in \mathbb{Z}^n \mid \sigma(s, t) = 1 \text{ for all } t \in \mathbb{Z}^n\}$. Let $b_1, \ldots, b_n$ be a $\mathbb{Z}$-basis for $S$, and let $B$ denote the $n \times n$ matrix whose $i$th row is $b_i$. We call the basis $b_1, \ldots, b_n$ positive diagonal if $B$ is a diagonal matrix whose diagonal entries are all positive.

Using this terminology, the authors state and prove their main result. In particular, if $b_1, \ldots, b_n$ is a $\mathbb{Z}$-basis for $S$, and $z_i = x^{b_i}$ for each index $i$, then the following conditions are equivalent:

(i) $b_1, \ldots, b_n$ positive diagonal basis for $S$ (after reordering, if necessary),
(ii) $Z(L)$ is a commutative Laurent series ring over $k$ in $z_1, \ldots, z_n$ and
(iii) $Z(R)$ is a commutative power series ring over $k$ in $z_1, \ldots, z_n$.

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MSC:

16W60 Valuations, completions, formal power series and related constructions (associative rings and algebras)
16L30 Noncommutative local and semilocal rings, perfect rings
16S34 Group rings

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References:


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