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Twists and braids for general 3-fold flops. (English) Zbl 1431.14031
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Let $f : X \to X_{\text{con}}$ be a algebraic 3-fold flopping contraction with X having Gorenstein terminal singularities. By [*T. Bridgeland*, Invent. Math. 147, No. 3, 613–632 (2002; Zbl 1085.14017)] and [*J.-C. Chen*, J. Differ. Geom. 61, No. 2, 227–261 (2002; Zbl 1090.14003)], X and its flop is derived equivalent, and one can construct braid group action on the bounded derived category $D(X)$ of coherent sheaves X by braid relations of flop functors in some specific situation. This paper establishes a general construction of higher degree braid group action on $D(X)$.

As explained in the introduction of the paper, the main results are divided intro three parts. As for the first part, assume that in the contraction f each of the n irreducible exceptional curves is individually floppable. Then by [*M. Wemyss*, Invent. Math. 211, No. 2, 435–521 (2018; Zbl 1390.14012)], the flop functors are controlled by a real hyperplane arrangement $\mathcal{H}$ which need not be Coxeter. In Theorem 3.23, it is shown that $D(X)$ has an action of the fundamental group of the complexified complement of $\mathcal{H}$ by the flop functors. Thus one finds actions of higher degree braid group on $D(X)$.

The second part introduces the notion of J-twists, which is used to give a geometric description of generators of the action constructed, and is also used to study the case when the exceptional curves of f are not individually floppable. For each subset J of the curves in the fibers of f, the J-twist is an autoequivalence of $D(X)$ corresponding to monodromy around a wall of codimension $|J|$. This autoequivalence is constructed by the noncommutative algebra A_J representing the functor of simultaneous noncommutative deformations of the reduced fibers in J. Explicitly, A_J is the quotient of the noncommutative algebra A by idempotents corresponding to the curves in J, where A is the one derived equivalent to the formal fiber of f [*M. Van den Bergh*, Duke Math. J. 122, No. 3, 423–455 (2004; Zbl 1074.14013)]. See Theorem 5.23 for the precise statement.

The third part gives another new construction of autoequivalences, called fiber twists, using commutative deformations of exceptional fibers. Similarly as J-twists, fiber twists are also constructed from the quotient A_0 of A by all the idempotents. See Theorem 5.23 for the precise statement. Fiber twists does not belong to the subgroup of autoequivalences generated by J-twists. In the type A case, these two twists are conjugate by line bundle twist, as shown in Theorem 6.5.

Both the construction of J-twists and fiber twists and the proof that they are twist autoequivalences are based on the theory of mutations. In particular, the construction of fiber twists is based on the maximal mutation algebras developed in [*O. Iyama* and *M. Wemyss*, Invent. Math. 197, No. 3, 521–586 (2014; Zbl 1308.14007)]. This article gives a brief recollection and an excellent application of these topics.

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MSC:

14J30	3-folds
14E30	Minimal model program (Mori theory, extremal rays)
14F06	Sheaves in algebraic geometry
16S38	Rings arising from noncommutative algebraic geometry
20F36	Braid groups; Artin groups
52C35	Arrangements of points, flats, hyperplanes (aspects of discrete geometry)

Keywords:
birational geometry; complex 3-fold; flops; rational curve; contraction algebra; Dynkin type; homological algebra; derived category; autoequivalence; braid-type group; hyperplane arrangement; Deligne groupoid

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