In the present paper, the author constructs toroidal compactifications for integral models of Shimura varieties of Hodge type. More precisely, he works with Shimura data \((G, X)\) that admit embeddings into a Siegel Shimura datum \((\text{GSp}(V), S^\pm(V))\) attached to a symplectic space \(V\) over \(\mathbb{Q}\). His results reduce the problem to understanding the integral models and cover all previously known cases of PEL type. In particular, at primes where the level is hyperspecial, his compactifications are shown to be canonical in a precise sense. The author’s method is to work locally, using \(p\)-adic Hodge theory and some notions from basic rigid analytic geometry. In general, for any class of Shimura varieties of Hodge type, once one has a reasonable theory of normal integral models, his results will supply good compactifications. As applications, he gives a new rationality property for Hodge cycles on abelian varieties with respect to \(p\)-adic uniformizations, and a different proof of Y. Morita’s conjecture: Suppose that \(A\) is an abelian variety defined over a number field and suppose that its Mumford-Tate group is anisotropic modulo its center. Then \(A\) has potential good reduction.

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MSC:

14M27 Compactifications; symmetric and spherical varieties
11G18 Arithmetic aspects of modular and Shimura varieties
14G35 Modular and Shimura varieties

Keywords:
Shimura varieties; compactifications; abelian varieties; logarithmic Dieudonné theory

Full Text: DOI arXiv Link