Bownik, Marcin; Casazza, Pete; Marcus, Adam W.; Speegle, Darrin
Improved bounds in Weaver and Feichtinger conjectures. (English) [Zbl 1432.46037]


The resolution of the Kadison-Singer problem raises the question of finding optimal quantitative bounds in these equivalent problems. In the first part, the authors sharpen the constant in Weaver’s KS\(_2\) conjecture that was given by Marcus et al. [loc. cit.] and show the following: For every \(\eta > 4\), there exists \(\vartheta > 0\) such that the following holds. Let \(u_1, \ldots, u_m \in \mathbb{C}^d\) be such that \(\|u_i\| \leq 1\) for all \(i\) and

\[
\sum_{i=1}^{m} |\langle u, u_i \rangle|^2 = \eta \quad \text{for all } \|u\| = 1.
\]

Then there exists a partition of \(\{1, \ldots, m\}\) into sets \(I_1\) and \(I_2\) so that, for \(k = 1, 2\),

\[
\sum_{i \in I_k} |\langle u, u_i \rangle|^2 \leq \eta - \vartheta \quad \text{for all } \|u\| = 1.
\]

This improves the original work that yields the same result albeit for constants \(\eta > (2 + \sqrt{2})^2\).

In the second part, the authors prove optimal asymptotic bounds on the size of the partitions in the Feichtinger conjecture.

Reviewer: Simon Lücking (Basel)

MSC:
46L05 General theory of \(C^*\)-algebras
46C05 Hilbert and pre-Hilbert spaces: geometry and topology (including spaces with semidefinite inner product)

Keywords:
Kadison-Singer problem; Weaver conjecture; Feichtinger conjecture

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References: