Two first-order logics of permutations. (English) Zbl 1433.05004

Summary: We consider two orthogonal points of view on finite permutations, seen as pairs of linear orders (corresponding to the usual one line representation of permutations as words) or seen as bijections (corresponding to the algebraic point of view). For each of them, we define a corresponding first-order logical theory, that we call TOTO (Theory Of Two Orders) and TOOB (Theory Of One Bijection) respectively. We consider various expressibility questions in these theories.

Our main results go in three different directions. First, we prove that, for all $k \geq 1$, the set of $k$-stack sortable permutations in the sense of West is expressible in TOTO, and that a logical sentence describing this set can be obtained automatically. Previously, descriptions of this set were only known for $k \leq 3$.

Next, we characterize permutation classes inside which it is possible to express in TOTO that some given points form a cycle. Lastly, we show that sets of permutations that can be described both in TOOB and TOTO are in some sense trivial. This gives a mathematical evidence that permutations-as-bijections and permutations-as-words are somewhat different objects.

MSC:
05A05 Permutations, words, matrices
03B10 Classical first-order logic

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permutations; patterns; first-order logic; Ehrenfeucht-Fraïssé games; sorting operators

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References:
[16] Karpilovskij, M., Composability of permutation classes (2017), arXiv preprint · Zbl 1409.05009
[18] Lehtonen, E., Permutation groups arising from pattern involvement (2016), arXiv preprint

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