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No lattice tiling of $\mathbb{Z}^n$ by Lee sphere of radius 2. (English) Zbl 1433.05148

Summary: We prove the nonexistence of lattice tilings of $\mathbb{Z}^n$ by Lee spheres of radius 2 for all dimensions $n \geq 3$. This implies that the Golomb-Welch conjecture [S. W. Golomb and L. R. Welch, SIAM J. Appl. Math. 18, 302–317 (1970; Zbl 0192.56302)] is true when the common radius of the Lee spheres equals 2 and $2n^2 + 2n + 1$ is a prime. As a direct consequence, we also answer an open question in the degree-diameter problem of graph theory: the order of any abelian Cayley graph of diameter 2 and degree larger than 5 cannot meet the abelian Cayley Moore bound.

MSC:
05C25 Graphs and abstract algebra (groups, rings, fields, etc.)
94B05 Linear codes (general theory)
05B45 Combinatorial aspects of tessellation and tiling problems

Keywords:
Golomb-Welch conjecture; lattice tiling; algebraic tiling; degree-diameter problem; perfect Lee code

Full Text: DOI arXiv

References:

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