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Asymptotic enumeration of compacted binary trees of bounded right height. (English)


Summary: A compacted binary tree is a graph created from a binary tree such that repeatedly occurring subtrees in the original tree are represented by pointers to existing ones, and hence every subtree is unique. Such representations form a special class of directed acyclic graphs. We are interested in the asymptotic number of compacted trees of given size, where the size of a compacted tree is given by the number of its internal nodes. Due to its superexponential growth this problem poses many difficulties. Therefore we restrict our investigations to compacted trees of bounded right height, which is the maximal number of edges going to the right on any path from the root to a leaf.

We solve the asymptotic counting problem for this class as well as a closely related, further simplified class.

For this purpose, we develop a calculus on exponential generating functions for compacted trees of bounded right height and for relaxed trees of bounded right height, which differ from compacted trees by dropping the above described uniqueness condition. This enables us to derive a recursively defined sequence of differential equations for the exponential generating functions. The coefficients can then be determined by performing a singularity analysis of the solutions of these differential equations. Our main results are the computation of the asymptotic numbers of relaxed as well as compacted trees of bounded right height and given size, when the size tends to infinity.

MSC:
05C30  Enumeration in graph theory
05C05  Trees
05C20  Directed graphs (digraphs), tournaments
05A16  Asymptotic enumeration

Keywords:
compacted trees; enumeration; d-finiteness; analytic combinatorics; directed acyclic graphs; Chebyshev polynomials

Software:
DLMF

Full Text: DOI arXiv Link

References:


Flajolet, P.; Sedgewick, R., Analytic Combinatorics (2009), Cambridge University Press · Zbl 1155.05001


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