Summary: A celebrated result of D. A. Barrington [Width-3 permutation branching programs. Techn. Rep. MIT/LCS/TM-29, Massachusetts Institute of Technology (1985)] proved that polynomial size, width-5 branching programs (BP) are equivalent in power to a restricted form of branching programs – polynomial sized width-5 permutation branching programs (PBP), which in turn capture all of NC1. On the other hand it is known that width-3 PBPs require exponential size to compute the AND function. No such lower bound is known for width-4 PBPs, however it is widely conjectured that width-4 PBPs will not capture all of NC1. In this work, we study the power of bounded width branching programs by comparing them with bounded width skew circuits.

It is well known that branching programs of bounded width have the same power as skew circuit of bounded width. The naïve approach converts a BP of width \( w \) to a skew circuit of width \( w^2 \). We improve this bound and show that BP of width \( w \geq 5 \) can be converted to a skew circuit of width 7. This also implies that skew circuits of bounded width are equal in power to skew circuits of width 7. For the other way, we prove that for any \( w \geq 2 \), a skew circuit of width \( w \) can be converted into an equivalent branching program of width \( w \). We prove that width-2 skew circuits are not universal while width-3 skew circuits are universal and that any polynomial sized CNF or DNF is computable by width 3 skew circuits of polynomial size. It is known that Parity does not have small CNFs or DNFs. It is easy to see that Parity has width-4 skew circuits.

We prove that a width-3 skew circuit computing Parity requires exponential size. This gives an exponential separation between the power of width-3 skew circuits and width-4 skew circuits.

MSC:

68Q06 Networks and circuits as models of computation; circuit complexity
68P05 Data structures
68Q17 Computational difficulty of problems (lower bounds, completeness, difficulty of approximation, etc.)

Keywords:

branching programs; Barrington’s theorem; skew circuits; lower bounds; parity

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References:
