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Integral Cayley graphs. (English. Russian original) Zbl 1434.05069
Algebra Logic 58, No. 4, 297-305 (2019); translation from Algebra Logika 58, No. 4, 445-457 (2019).

Summary: Let $G$ be a group and $S \subseteq G$ a subset such that $S = S^{-1}$, where $S^{-1} = \{s^{-1} | s \in S\}$. Then a Cayley graph $\text{Cay}(G, S)$ is an undirected graph $\Gamma$ with vertex set $V(\Gamma) = G$ and edge set $E(\Gamma) = \{(g, gs) | g \in G, s \in S\}$. For a normal subset $S$ of a finite group $G$ such that $s \in S \Rightarrow s^k \in S$ for every $k \in \mathbb{Z}$ which is coprime to the order of $s$, we prove that all eigenvalues of the adjacency matrix of $\text{Cay}(G, S)$ are integers. Using this fact, we give affirmative answers to Questions 19.50(a) and 19.50(b) in the [V. D. Mazurov (ed.) and E. I. Khukhro (ed.), The Kourovka notebook. Unsolved problems in group theory. 19th ed. Novosibirsk: Institute of Mathematics, Russian Academy of Sciences, Siberian Div. (2018)].

MSC:

05C25 Graphs and abstract algebra (groups, rings, fields, etc.)

05C50 Graphs and linear algebra (matrices, eigenvalues, etc.)

Keywords:

Cayley graph; adjacency matrix of graph; spectrum of graph; integral graph; complex group algebra; character of group

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References:


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