
Summary: We study uniform measures in the first Heisenberg group $\mathbb{H}$ equipped with the Korányi metric $d_H$. We prove that 1-uniform measures are proportional to the spherical 1-Hausdorff measure restricted to an affine horizontal line, while 2-uniform measures are proportional to spherical 2-Hausdorff measure restricted to an affine vertical line. We also show that each 3-uniform measure which is supported on a vertically ruled surface is proportional to the restriction of spherical 3-Hausdorff measure to an affine vertical plane, and that no quadratic $x_3^2$-graph can be the support of a 3-uniform measure. According to a result of Merlo, every 3-uniform measure is supported on a quadratic variety; in conjunction with our results, this shows that all 3-uniform measures are proportional to spherical 3-Hausdorff measure restricted to an affine vertical plane. We establish our conclusions by deriving asymptotic formulas for the measures of small extrinsic balls in $(\mathbb{H}, d_H)$ intersected with smooth submanifolds. The coefficients in our power series expansions involve intrinsic notions of curvature associated to smooth curves and surfaces in $\mathbb{H}$.

MSC:
28A78 Hausdorff and packing measures
53C17 Sub-Riemannian geometry

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References:

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