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Recognizing read-once functions from depth-three formulas. (English) Zbl 1434.68201

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Summary: Consider the following decision problem: for a given monotone Boolean function \( f \) decide, whether \( f \) is read-once. For this problem, it is essential how the input function \( f \) is represented. K. Elbassioni et al. [J. Comb. Optim. 22, No. 3, 293–304 (2011; Zbl 1229.90090)] proved that this problem is coNP-complete when \( f \) is given by a depth-4 read-2 monotone Boolean formula. V. Gurvich [It is a coNP-complete problem to decide whether a positive \( \lor \land \) formula of depth 3 defines a read-once or respectively quadratic Boolean function. Techn. Rep. RRR 4-2010, Center for Operations Research, Rutgers University (2010)] proved that this problem is coNP-complete even when the input is the following expression: \( C \lor D_n \), where \( D_n = x_1 y_1 \lor \cdots \lor x_n y_n \) and \( C \) is a monotone CNF over the variables \( x_1, y_1, \ldots, x_n, y_n \) (note that this expression is a monotone Boolean formula of depth 3; in Gurvich, loc. cit. nothing is said about the readability of \( C \), but the proof is valid even if \( C \) is read-2 and thus the entire formula is read-3). We show that we can test in polynomial-time whether a given expression \( C \lor D \) computes a read-once function, provided that \( C \) is a read-once monotone CNF and \( D \) is a read-once monotone DNF and all the variables of \( C \) occur also in \( D \) (recall that due to Gurvich, the problem is coNP-complete when \( C \) is read-2). We also observe that from the so-called Sausage Lemma of E. Boros et al. [in: Polyhedral computation. Papers presented at a workshop, Montréal, Canada, 2006. Providence, RI: American Mathematical Society (AMS). 15–43 (2009; Zbl 1170.68619)] it follows that the problem of recognizing read-once functions is coNP-complete when the input formula is depth-3 read-2.

MSC:

68Q25 Analysis of algorithms and problem complexity
06E30 Boolean functions
68Q17 Computational difficulty of problems (lower bounds, completeness, difficulty of approximation, etc.)

Keywords:
read-once functions; NP-completeness; monotone Boolean functions; depth-three formulas

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References:

[1] Boros, Endre; Elbassioni, Khaled; Gurvich, Vladimir; Makino, Kazuhisa, Generating vertices of polyhedra and related problems of monotone generation, Polyhedral Computation, 15-43 (2009), Providence, Rhode Island: American Mathematical Society, Providence, Rhode Island · Zbl 1170.68619
[7] Gurvich, V.: It is a cmp-complete problem to decide whether a positive \( \exists \exists \) formula of depth 3 defines a read-once or respectively quadratic boolean function. Rutcor Research Report (2010)

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