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A variational characterization of profile curves of invariant linear Weingarten surfaces.
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Summary: We show that the profile curve of any invariant linear Weingarten (LW) surface of a semi-Riemannian 3-space form, $M^3(\rho)$, is an extremal curve for a curvature energy. Moreover, we also give a construction procedure of LW surfaces from extremal curves of this energy, which allows us to understand invariant LW surfaces as binormal evolution surfaces with prescribed velocity. In particular, if the profile curve is planar, the variational problem depends on a total curvature type energy. We use this characterization to give a new approach to locally get all invariant constant Gaussian curvature surfaces whose profile curve is planar. Indeed, we also prove that the orbits of these surfaces are critical curves of the same energy acting on a different space of curves. Finally, restricting ourselves to the Riemannian case, $M^3(\rho)$, we study the existence of closed rotational surfaces with constant Gaussian curvature and we see that the planar profile curves of these rotational surfaces of $M^3(\rho)$ are just the horizontal projections of sub-Riemannian geodesics of semi-Riemannian unit tangent bundles.

MSC:
53C42 Differential geometry of immersions (minimal, prescribed curvature, tight, etc.)
53C50 Global differential geometry of Lorentz manifolds, manifolds with indefinite metrics
53C17 Sub-Riemannian geometry
53A05 Surfaces in Euclidean and related spaces

Keywords:
constant Gaussian curvature surfaces; extremal curves; linear Weingarten surfaces; sub-Riemannian geodesics; total curvature type energy; unit tangent bundles

Full Text: DOI

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