

Pandharipande, Rahul; Yin, Qizheng

Relations in the tautological ring of the moduli space of $K3$ surfaces. (English) Zbl 1436.14069
J. Eur. Math. Soc. (JEMS) 22, No. 1, 213–252 (2020).

The paper under review deals with the interplay of the moduli of curves and the moduli of $K3$ surfaces via the virtual class of the moduli spaces of stable maps. Using Getzler's relation for $\overline{M}_{1,4}$ [*E. Getzler*, *J. Am. Math. Soc.* 10, No. 4, 973–998 (1997; [Zbl 0909.14002](#))], the authors construct a universal decomposition of the diagonal in Chow in the third fiber product of the universal $K3$ surface. This decomposition together with the Witten-Dijkgraaf-Verlinde-Verlinde relation in genus 0 is used to prove a conjecture of Marian-Oprea-Pandharipande [*A. Marian et al.*, *Ann. Sci. Éc. Norm. Supér. (4)* 50, No. 1, 239–267 (2017; [Zbl 1453.14016](#))]: the full tautological ring of the moduli space of $K3$ surfaces is generated in Chow by the classes of the Noether-Lefschetz loci. Furthermore the authors propose a connection between relations in the tautological ring of the moduli spaces of curves and relations in the tautological ring of the moduli space of $K3$ surfaces.

Reviewer: [Mauro Fortuna \(Hannover\)](#)

MSC:

- [14J28](#) $K3$ surfaces and Enriques surfaces
- [14J10](#) Families, moduli, classification: algebraic theory
- [14C15](#) (Equivariant) Chow groups and rings; motives
- [14D07](#) Variation of Hodge structures (algebraic-geometric aspects)
- [14N35](#) Gromov-Witten invariants, quantum cohomology, Gopakumar-Vafa invariants, Donaldson-Thomas invariants (algebraic-geometric aspects)

Cited in 2 Documents

Keywords:

$K3$ surfaces; moduli spaces; tautological cycles; Noether-Lefschetz loci; Gromov-Witten theory

Full Text: [DOI](#) [arXiv](#)

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