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Authors’ abstract: Linear inverse problems $A\mu = y$ with Poisson noise and non-negative unknown $\mu \geq 0$ are ubiquitous in applications, for instance in positron emission tomography (PET) in medical imaging. The associated maximum likelihood problem is routinely solved using an expectation-maximisation algorithm (ML-EM). This typically results in images which look spiky, even with early stopping. We give an explanation for this phenomenon. We first regard the image $\mu$ as a measure. We prove that if the measurements $y$ are not in the cone $\{A\mu, \mu \geq 0\}$, which is typical of low injected dose, likelihood maximisers must be sparse, i.e., typically a sum of point masses. We also show a weak sparsity result for cluster points of ML-EM. On the other hand, in the low noise regime, we prove that cluster points of ML-EM are optimal measures with full support. Finally, we provide concentration bounds for the probability to be in the sparse case, and a set of numerical experiments supporting our claims.

Reviewer: Alessandro Selvitella (Fort Wayne)

MSC:
62L15 Optimal stopping in statistics
65J22 Numerical solution to inverse problems in abstract spaces
62H35 Image analysis in multivariate analysis
92C55 Biomedical imaging and signal processing
62P10 Applications of statistics to biology and medical sciences; meta analysis
62H30 Classification and discrimination; cluster analysis (statistical aspects)

Keywords:
maximum likelihood; inverse problems; expectation-maximisation; Kullback-Leibler divergence; positron emission tomography; Richardson-Lucy

Software:
MLEM; ODL; GitHub

Full Text: DOI arXiv

References:
[31] Pouchol C and Verdier O MLEM Experiment Notebook https://github.com/olivierverdier/ml_em_notebook
[34] Richardson W H 1972 Bayesian-based iterative method of image restoration JOSA62 55-9 · doi:10.1364/josa.62.00055

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