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The algebraic matroid of the finite unit norm tight frame (funtf) variety. (English)
Zbl 1437.05037

Summary: A finite unit norm tight frame is a collection of \( r \) vectors in \( \mathbb{R}^n \) that generalizes the notion of orthonormal bases. The affine finite unit norm tight frame variety is the Zariski closure of the set of finite unit norm tight frames. Determining the fiber of a projection of this variety onto a set of coordinates is called the algebraic finite unit norm tight frame completion problem. Our techniques involve the algebraic matroid of an algebraic variety, which encodes the dimensions of fibers of coordinate projections. This work characterizes the bases of the algebraic matroid underlying the variety of finite unit norm tight frames in \( \mathbb{R}^3 \). Partial results towards similar characterizations for finite unit norm tight frames in \( \mathbb{R}^n \) with \( n \geq 4 \) are also given. We provide a method to bound the degree of the projections based off of combinatorial data.

MSC:
05B35 Combinatorial aspects of matroids and geometric lattices
52B40 Matroids in convex geometry (realizations in the context of convex polytopes, convexity in combinatorial structures, etc.)
42C15 General harmonic expansions, frames

Keywords:
algebraic matroid; frame completion

Software:
PHCpack; Bertini; Traces; nauty; LamanGraphs; Macaulay2; Bertini.m2; Mathematica; NumericalAlgebraicGeometry

Full Text: DOI

References:
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