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Periodic expansiveness of smooth surface diffeomorphisms and applications. (English)

Zbl 1437.37052
J. Eur. Math. Soc. (JEMS) 22, No. 2, 413-454 (2020).

The author of this paper considers the relationship between the topological entropy and the growth rate of the number of periodic points for smooth dynamical systems. His results include new results about those relationships. It is known that the topological entropy needs not being equal to the exponential growth rate of the number of periodic points. *R. Bowen* [Trans. Am. Math. Soc. 154, 377–397 (1971; Zbl 0212.29103)] had proved equality between the two for certain expansive homeomorphisms (those with topologically transitive subshifts of finite type and Axiom A systems).

The main results are as follows. If f is a C^∞ diffeomorphism from a compact surface M to itself with positive topological entropy $h_{\mathrm{top}}(f)$, then for any δ in the interval $(0, h_{\mathrm{top}}(f))$ the set of saddle n-periodic points with Lyapunov exponents δ-away from zero has exponential growth rate in n equal to the topological entropy. Furthermore, these periodic points are equidistributed with respect to measures of maximal entropy.

Similarly, if f is a C^∞ interval map with positive topological entropy $h_{top}(f)$, then the set of repelling n-periodic points with the same constraints on Lyapunov exponents has exponential growth rate in n equal to the topological entropy. Here again the periodic points are equidistributed with respect to measures of maximal entropy.

The author's strategy in the proof of the main theorem is to use the concept of local exponential growth rate and a result of *A. Katok* [Publ. Math., Inst. Hautes Étud. Sci. 51, 137–173 (1980; Zbl 0445.58015)]. Any subset of periodic points with a growth rate that matches or exceeds the topological entropy, but has zero local growth rate, is equidistributed with respect to maximal measures. But its exponential growth rate is equal to the topological entropy. Local exponential growth rate is the exponential growth rate in n of the n-periodic points in an arbitrarily small n-dynamical ball. Using Katok's theorem, it is then only necessary to show that the set of saddle n-periodic points with Lyapunov exponents δ-away from zero (and the corresponding set for interval maps) have zero local exponential growth rates for C^∞ surface diffeomorphisms and interval maps.

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MSC:

37E30 Dynamical systems involving homeomorphisms and diffeomorphisms of planes and surfaces
37B40 Topological entropy
37E10 Dynamical systems involving maps of the circle
37C05 Dynamical systems involving smooth mappings and diffeomorphisms
37C25 Fixed points and periodic points of dynamical systems; fixed-point index theory; local dynamics
37D25 Nonuniformly hyperbolic systems (Lyapunov exponents, Pesin theory, etc.)
28D20 Entropy and other invariants
37C40 Smooth ergodic theory, invariant measures for smooth dynamical systems
14P10 Semialgebraic sets and related spaces

Keywords:
entropy; hyperbolic periodic points; smooth surface dynamical systems; Yomdin's theory; semi-algebraic geometry

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