Summary: The main result of this paper is a construction of solutions to the reverse Yang-Mills-Higgs flow converging in the $C^\infty$ topology to a critical point. The construction uses only the complex gauge group action, which leads to an algebraic classification of the isomorphism classes of points in the unstable set of a critical point in terms of a filtration of the underlying Higgs bundle.

Analysing the compatibility this filtration with the canonical Harder-Narasimhan-Seshadri double filtration gives an algebraic criterion for two critical points to be connected by a flow line. As an application, we use this to construct Hecke modifications of Higgs bundles via the Yang-Mills-Higgs flow. When the Higgs field is zero (corresponding to the Yang-Mills flow), this criterion has a geometric interpretation in terms of secant varieties of the projectivisation of the underlying bundle inside the unstable manifold of a critical point, which gives a precise description of broken and unbroken flow lines connecting two critical points. For non-zero Higgs field, at generic critical points the analogous interpretation involves the secant varieties of the spectral curve of the Higgs bundle.

MSC:

53C07 Special connections and metrics on vector bundles (Hermite-Einstein, Yang-Mills)
53E30 Flows related to complex manifolds (e.g., Kähler-Ricci flows, Chern-Ricci flows)
58E15 Variational problems concerning extremal problems in several variables; Yang-Mills functionals

Keywords:
Higgs bundles; broken flow lines; secant varieties; spectral curve; gradient flow; upper and lower critical sets; Hecke correspondence

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