Filipkovskaya, M. S.

A block form of a singular pencil of operators and a method of obtaining it. (Russian. English summary) Zbl 1438.47027

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Summary: A block form of a singular operator pencil $\lambda A + B$, where λ is a complex parameter, and the linear operators A, B act in finite-dimensional spaces, is described. An operator pencil $\lambda A + B$ is called regular if $n = m = rk(\lambda A + B)$, where $rk(\lambda A + B)$ is the rank of the pencil and m, n are the dimensions of spaces (the operators map an n-dimensional space into an m-dimensional one); otherwise, if $n \neq m$ or $n = m$ and $rk(\lambda A + B) < n$, the pencil is called singular (irregular). The block form (structure) consists of a singular block, which is a purely singular pencil, i.e., it is impossible to separate out a regular block in this pencil, and a regular block. In these blocks, zero blocks and blocks which are invertible operators are separated out. A method of obtaining the block form of a singular operator pencil is described in detail for two special cases, when $rk(\lambda A + B) = m < n$ and $rk(\lambda A + B) = n < m$, and for the general case, when $rk(\lambda A + B) < n, m$. Methods for the construction of projectors onto subspaces from the direct decompositions, relative to which the pencil has the required block form, are given. Using these projectors, we can find the form of the blocks and, accordingly, the block form of the pencil. Examples of finding the block form for various types of singular pencils are presented. To obtain the block form, in particular, the results regarding the reduction of a singular pencil of matrices to the canonical quasidiagonal form, which is called the Weierstrass-Kronecker canonical form, are used. Also, methods of linear algebra are used. The obtained block form of the pencil and the corresponding projectors can be used to solve various problems. In particular, it can be used to reduce a singular semilinear differential-operator equation to the equivalent system of purely differential and purely algebraic equations. This greatly simplifies the analysis and solution of differential-operator equations.

MSC:

47A56 Functions whose values are linear operators (operator- and matrix-valued functions, etc., including analytic and meromorphic ones)
15A22 Matrix pencils
47N20 Applications of operator theory to differential and integral equations

Keywords:

operator pencil; matrix pencil; singular; regular block; block form

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