Wu, Bing-Ling; Chen, Yong-Gao
On the denominators of harmonic numbers. (Sur les dénominateurs des nombres harmoniques.) (English. French summary) Zbl 1439.11078

For a positive integer \( n \), write the \( n \)-th harmonic number \( H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} \) as \( \frac{u_n}{v_n} \) with relatively prime positive integers \( u_n \) and \( v_n \). For a prime number \( p \), define the sets \( J_p = \{ n : p \mid u_n \} \) and \( I_p = \{ n : p \nmid v_n \} \).

It is clear that \( J_p \subseteq I_p \). A. Eswarathasan and E. Levine [Discrete Math. 91, No. 3, 249–257 (1991; Zbl 0764.11018)] conjectured that for every prime number \( p \), \( J_p \) is finite. D. W. Boyd [Exp. Math. 3, No. 4, 287–302 (1994; Zbl 0838.11015)] verified this conjecture up to 547, with three exceptions: \( p = 83, 127, 397 \).

The main result of the paper supports the conjecture: for every positive integer \( m \), the set \( \{ n : m \mid v_n \} \) has density one.

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MSC:
11B75 Other combinatorial number theory
11B05 Density, gaps, topology
11B83 Special sequences and polynomials

Keywords:
harmonic numbers; density one; denominator

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References:

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