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Long time oscillation of solutions of nonlinear Schrödinger equations near minimal mass ground state. (English) Zbl 1439.35439

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Summary: In this paper, we consider the long time dynamics of radially symmetric solutions of nonlinear Schrödinger equations (NLS) having a minimal mass ground state. In particular, we show that there exist solutions with initial data near the minimal mass ground state that oscillate for long time. More precisely, we introduce a coordinate defined near the minimal mass ground state which consists of finite and infinite dimensional part associated to the discrete and continuous part of the linearized operator. Then, we show that the finite dimensional (2-D) part approximately obeys Newton's equation of motion for a particle in an anharmonic potential well. Showing that the infinite dimensional part is well separated from the finite dimensional part, we will have long time oscillation.

MSC:

35Q55 NLS equations (nonlinear Schrödinger equations)

35B06 Symmetries, invariants, etc. in context of PDEs

35B05 Oscillation, zeros of solutions, mean value theorems, etc. in context of PDEs

35B40 Asymptotic behavior of solutions to PDEs

Cited in **3** Documents

Keywords:

nonlinear Schrödinger equations; oscillations

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