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Convergence of mean curvature flow in hyper-Kähler manifolds. (English) Zbl 1439.53080

Summary: Inspired by work of N. C. Leung and T. Y. H. Wan [J. Geom. Anal. 17, No. 2, 343–364 (2007; Zbl 1118.53027)], we study the mean curvature flow in hyper-Kähler manifolds starting from hyper-Lagrangian submanifolds, a class of middle-dimensional submanifolds, which contains the class of complex Lagrangian submanifolds. For each hyper-Lagrangian submanifold, we define a new energy concept called the twistor energy by means of the associated twistor family (i.e., 2-sphere of complex structures). We show that the mean curvature flow starting at any hyper-Lagrangian submanifold with sufficiently small twistor energy exists for all time and converges to a complex Lagrangian submanifold for one of the hyper-Kähler complex structure. In particular, our result implies some kind of energy gap theorem for hyper-Kähler manifolds which have no complex Lagrangian submanifolds.

MSC:
53E10 Flows related to mean curvature
53C26 Hyper-Kähler and quaternionic Kähler geometry, “special” geometry

Keywords:
mean curvature flow; hyper-Kähler manifolds; hyper-Lagrangian submanifolds; twistor energy; energy gap

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