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A dependence with complete connections approach to generalized Rényi continued fractions.
(English) Zbl 1440.11126

The authors study continued fractions associated with the one parameter family of interval maps of the form
\[ T_u(x) = \frac{1}{u(1-x)} - \left\lfloor \frac{1}{u(1-x)} \right\rfloor, \]
where \( u > 0, x \in [0,1) \), \( \lfloor \cdot \rfloor \) denotes the floor function, for the values \( u = 1/N, N \geq 2 \) an integer. They refer to this continued fraction as Rényi-type continued fraction and \( T_{1/N} \) is indicated by \( R_N \).

The transformation \( T_u \) was addressed by K. Gröchenig and A. Haas [Ergodic Theory Dyn. Syst. 16, No. 6, 1241–1274 (1996; Zbl 0884.58040)]. These, and general related transformatins, have been looked into with regard to dynamical properties and lead to a wide range of connections between continued fractions, diophantine approximation, ergodic theory and hyperbolic geometry, cf. the Introduction section in the paper under review.

The Rényi-type continued fraction transformation \( R_N[0,1] \rightarrow [0,1] \) is defined by
\[ R_N(x) = \begin{cases} \frac{1}{u(1-x)} - \left\lfloor \frac{1}{u(1-x)} \right\rfloor, & x \in [0,1), \\ 0, & x = 1, \end{cases} \]
and the associated continued fraction is given by
\[ x = 1 - \frac{N}{1 + a_1} - \frac{N}{1 + a_2} - \frac{N}{1 + a_3} - \cdots = [a_1, a_2, a_3, \cdots]_{R}, \]
where the digits \( a_n(x), n \in \mathbb{N}_+ \) are defined as
\[ \begin{cases} a_n = a_n(x) = a_1 \left( R_N^{n-1}(x) \right), & n \geq 1 \ (R_N^0(x) = x) \\ a_1 = a_1(x) = \begin{cases} \frac{N}{1-x} & \text{if } x \neq 1, \\ \infty & \text{if } x = 1. \end{cases} \end{cases} \]

This continued fraction can be seen as a measure preserving dynamical system \( ([0,1], \mathcal{B}_{[0,1]}, R_N, \rho_N) \) with \( \mathcal{B}_{[0,1]} \) the \( \sigma \)-algebra of Borel subsets of \([0,1]\) and
\[ \rho_N(A) = \frac{1}{\log \left( \frac{N}{N-1} \right)} \int_A \frac{dx}{x + N-1}, \quad A \in \mathcal{B}_{[0,1]} \]
is the invariant probability measure under \( R_N \).

The layout of the paper is as follows:
§1. Introduction (2 pages)
§2. Rényi-type continued fraction expansions as dynamical system (2 pages)
§3. The probabilistic structure of \( \{a_n\}_{n \in \mathbb{N}_+} \) under the Lebesgue measure (2 pages)
Results are Proposition 3.1 (Brodén-Borel-Lévy-type formula) and Proposition 3.2 (the probabilistic structure of the digits \( a_n \) under the lebesgue measure on \([0,1]\)).
§4. Natural extension and extended random variables (3 pages)
Main result Theorem 4.3 (featuring the natural extension \( \overline{R}_N \) and \( \overline{\rho}_N \) to \([0,1]^2 \) with the Borel sets on the unit square).
§5. Perron-Frobenius operators (2 pages)
Defined through the Radon-nikodym theorem as the unique linear and positive operator on the Banach space $L^1([0,1]), \mu$ for a specific probability measure $\mu$.

§6. Random systems with complete connections and the Gauss-Kuzmin-type problem (8 pages)
This type of system is often called an iterated function system; main results in Theorem 6.6 (associated Markov chain being a Doeblin-Fortet operator), Theorem 6.7 (an a compact Markov chain being ordered and the existence of a special transition probability function), Theorem 6.16 (a Gauss-Kuzmin-type theorem for $R_N$).

References (contains 23 items)

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MSC:
11J70 Continued fractions and generalizations
28D05 Measure-preserving transformations
37A30 Ergodic theorems, spectral theory, Markov operators
60A10 Probabilistic measure theory

Keywords:
Rényi continued fraction; Perron-Frobenius operator; random system with complete connections; Gauss-Kuzmin problem

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