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**Arithmetic and representation theory of wild character varieties.** (English) Zbl 1440.14234  
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**Summary:** We count points over a finite field on wild character varieties of Riemann surfaces for singularities with regular semisimple leading term. The new feature in our counting formulas is the appearance of characters of Yokonuma-Hecke algebras. Our result leads to the conjecture that the mixed Hodge polynomials of these character varieties agree with the previously conjectured perverse Hodge polynomials of certain twisted parabolic Higgs moduli spaces, indicating the possibility of a  $P = W$  conjecture for a suitable wild Hitchin system.

**MSC:**

- 14M35 Character varieties
- 14D20 Algebraic moduli problems, moduli of vector bundles
- 14G05 Rational points
- 14H60 Vector bundles on curves and their moduli
- 20G05 Representation theory for linear algebraic groups

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**Keywords:**

irregular connection; character variety; Hitchin system; Yokonuma-Hecke algebra; Macdonald polynomials

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