The authors investigate critical points and minimizers of the Yang-Mills functional $Y_M$ on quantum Heisenberg manifolds $D_{\mu\nu}$. A Yang-Mills connection is a compatible linear connection which is both a critical point and a minimizer of the Yang-Mills functional $Y_M$. In this paper, the authors investigate Yang-Mills connections with constant curvature. Connections on the following classes of modules are considered: (i) Abadies’s module $\Xi$ of trace $2\mu$ and its submodules, (ii) modules $\Xi'$ of trace $2\nu$, (iii) tensor product modules of the form $PE_{\mu\nu}^c \otimes \Xi$, where $E_{\mu\nu}^c$ is Morita equivalent to $C$ and $P$ is a projection in $E_{\mu\nu}^c$.

The authors show that every Yang-Mills connection $\nabla$ on $\Xi$ over $D_{\mu\nu}$ with constant curvature has curvature of the form $\Theta\nabla(X, Y) = \Theta\nabla(X, Z) = 0$ and $\Theta\nabla(Y, Z) = \pi_\mu Id_E$.

Also, they show that Yang-Mills connections with constant curvature do not provide global minima but only local minima, which is done by constructing a set of compatible connections that are not critical points but whose values are smaller than those of Yang-Mills connections with constant curvature. Other results of the authors include: (i) an example of a compatible linear connection with constant curvature on $D_{\mu\nu}$ such that the corresponding connection on an isomorphic module does not have constant curvature, and (ii) the construction of a compatible linear connection with constant curvature which neither attains its minimum nor is a critical point of $Y_M$ on $D_{\mu\nu}$. Furthermore, the authors construct: (iii) the Grassmannian connection on the projective modules $\Xi'$ with trace $2\nu$ over $D_{\mu\nu}$ and compute its corresponding curvature, and (iv) tensor product connections on $PE_{\mu\nu}^c \otimes \Xi$ whose coupling constant is $2\nu$ and characterize the critical points of $Y_M$ for this projective module.

Reviewer: Jan Kurek (Lublin)

MSC:

53C07 Special connections and metrics on vector bundles (Hermite-Einstein, Yang-Mills)
58E15 Variational problems concerning extremal problems in several variables; Yang-Mills functionals
81T13 Yang-Mills and other gauge theories in quantum field theory

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