

Cais, Bryden

The geometry of Hida families. II: Λ -adic (φ, Γ) -modules and Λ -adic Hodge theory. (English)

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Summary: We construct the Λ -adic crystalline and Dieudonné analogues of Hida's ordinary Λ -adic étale cohomology, and employ integral p -adic Hodge theory to prove Λ -adic comparison isomorphisms between these cohomologies and the Λ -adic de Rham cohomology studied in [the author, Math. Ann. 372, No. 1–2, 781–844 (2018; Zbl 1441.11098)] as well as Hida's Λ -adic étale cohomology. As applications of our work, we provide a 'cohomological' construction of the family of (φ, Γ) -modules attached to Hida's ordinary Λ -adic étale cohomology by *J. Dee* [J. Algebra 235, No. 2, 636–664 (2001; Zbl 0984.11062)], and we give a new and purely geometric proof of Hida's finiteness and control theorems. We also prove suitable Λ -adic duality theorems for each of the cohomologies we construct.

MSC:

11F33 Congruences for modular and p -adic modular forms

11F67 Special values of automorphic L -series, periods of automorphic forms, cohomology, modular symbols

11G18 Arithmetic aspects of modular and Shimura varieties

11R23 Iwasawa theory

14F30 p -adic cohomology, crystalline cohomology

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Keywords:

Hida families; integral p -adic Hodge theory; de Rham cohomology; crystalline cohomology

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