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The geometry of Hida families I: Λ-adic de Rham cohomology. (English) Zbl 1441.11098

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Summary: We construct the Λ-adic de Rham analogue of Hida's ordinary Λ-adic étale cohomology and of Ohta's Λ-adic Hodge cohomology, and by exploiting the geometry of integral models of modular curves over the cyclotomic extension of $\mathbb{Q}_p$, we give a purely geometric proof of the expected finiteness, control, and Λ-adic duality theorems. Following *M. Ohta* [J. Reine Angew. Math. 463, 49–98 (1995; Zbl 0827.11025)], we then prove that our Λ-adic module of differentials is canonically isomorphic to the space of ordinary Λ-adic cuspforms. In the sequel [the author, Compos. Math. 154, No. 4, 719–760 (2018; Zbl 1441.11097)] to this paper, we construct the crystalline counterpart to Hida's ordinary Λ-adic étale cohomology, and employ integral p-adic Hodge theory to prove Λ-adic comparison isomorphisms between all of these cohomologies. As applications of our work in this paper and [the author, loc. cit.], we will be able to provide a "cohomological" construction of the family of (φ,Γ)-modules attached to Hida's ordinary Λ-adic étale cohomology by *J. Dee* [J. Algebra 235, No. 2, 636–664 (2001; Zbl 0984.11062)], as well as a new and purely geometric proof of Hida's finiteness and control theorems. We are also able to prove refinements of the main theorems in *B. Mazur* and *A. Wiles* [Compos. Math. 59, 231–264 (1986; Zbl 0654.12008)] and Ohta [loc. cit.].

MSC:

11F33	Congruences for modular and p-adic modular forms	Cited in **1** Review
11F67	Special values of automorphic L-series, periods of automorphic forms, cohomology, modular symbols	
11G18	Arithmetic aspects of modular and Shimura varieties	
11R23	Iwasawa theory	

Keywords:

Λ-adic de Rham cohomology; Hida families; duality theorems

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