Antunez, Andrea C.
Sphere and projective space of a $C^*$-algebra with a faithful state. (English) [Zbl 1441.46046] Demonstr. Math. 52, 410-427 (2019).

Summary: Let $A$ be a unital $C^*$-algebra with a faithful state $\varphi$. We study the geometry of the unit sphere $S_\varphi = \{ x \in A : \varphi(x^*x) = 1 \}$ and the projective space $P_\varphi = S_\varphi / T$. These spaces are shown to be smooth manifolds and homogeneous spaces of the group $U_\varphi(A)$ of isomorphisms acting in $A$ which preserve the inner product induced by $\varphi$, which is a smooth Banach-Lie group. An important role is played by the theory of operators in Banach spaces with two norms, as developed by M. G. Krein [Integral Equations Oper. Theory 30, No. 2, 140–162 (1998; Zbl 0914.47002)] and P. D. Lax [Commun. Pure Appl. Math. 7, 633–647 (1954; Zbl 0057.34402)]. We define a metric in $P_\varphi$, and prove the existence of minimal geodesics, both with given initial data, and given endpoints.

MSC:
46L30 States of selfadjoint operator algebras
58B20 Riemannian, Finsler and other geometric structures on infinite-dimensional manifolds

Keywords:
homogeneous space; minimal curves; $C^*$-algebra; projective space

Full Text: DOI

References:

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