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**Group variable selection via  $\ell_{p,0}$  regularization and application to optimal scoring.** (English)

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Summary: The need to select groups of variables arises in many statistical modeling problems and applications. In this paper, we consider the  $\ell_{p,0}$ -norm regularization for enforcing group sparsity and investigate a DC (Difference of Convex functions) approximation approach for solving the  $\ell_{p,0}$ -norm regularization problem. We show that, with suitable parameters, the original and approximate problems are equivalent. Considering two equivalent formulations of the approximate problem we develop DC programming and DCA (DC Algorithm) for solving them. As an application, we implement the proposed algorithms for group variable selection in the optimal scoring problem. The sparsity is obtained by using the  $\ell_{p,0}$ -regularization that selects the same features in all discriminant vectors. The resulting sparse discriminant vectors provide a more interpretable low-dimensional representation of data. The experimental results on both simulated datasets and real datasets indicate the efficiency of the proposed algorithms.

**MSC:**

62F07 Statistical ranking and selection procedures

93E20 Optimal stochastic control

62P10 Applications of statistics to biology and medical sciences; meta analysis

Cited in 5 Documents

**Keywords:**

$\ell_{p,0}$  regularization; DC (Difference of Convex functions) approximation; DC programming; DCA; optimal scoring

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