Panin, I. A.; **Walter, C.**

On the relation of symplectic algebraic cobordism to Hermitian K-theory. (English. Russian original) Zbl 1442.19012

Proc. Steklov Inst. Math. 307, 162-173 (2019); translation from Tr. Mat. Inst. Steklova 307, 180-192 (2019).

Summary: We reconstruct hermitian K-theory via algebraic symplectic cobordism. In the motivic stable homotopy category $SH(S)$, there is a unique morphism $\varphi : \mathbf{MSp} \to \mathbf{BO}$ of commutative ring T-spectra which sends the Thom class $\mathrm{th}^{\mathbf{MSp}}$ to the Thom class $\mathrm{th}^{\mathbf{BO}}$. Using φ we construct an isomorphism of bi-graded ring cohomology theories on the category $\mathcal{S}m\mathcal{O}p/S$, $\bar\varphi : \mathbf{MSp}^{*,*}(X,U) \otimes_{\mathbf{MSp}^{4*,0*}(\mathrm{pt})} \mathrm{BO}^{4*,2*}(\mathrm{pt}) \cong \mathrm{BO}^{*,*}(X,U)$. The result is an algebraic version of the theorem of Conner and Floyd reconstructing real K-theory using symplectic cobordism. Rewriting the bigrading as $\mathbf{MSp}^{p,q} = \mathbf{MSp}^{[q]}_{1q-p}$, we have an isomorphism $\bar\varphi : \mathbf{MSp}^{[*]}_{*}(X,U) \otimes_{\mathbf{MSp}_0^{[2*]}(\mathrm{pt})} \mathrm{KO}_0^{[2*]}(\mathrm{pt}) \cong \mathrm{KO}^{[*]}_{*}(X,U)$, where the $KO_i^{[n]}(X,U)$ are Schlichting's hermitian K-theory groups.

MSC:

19G38	Hermitian K-theory, relations with K-theory of rings
55N22	Bordism and cobordism theories and formal group laws in algebraic topology
14F42	Motivic cohomology; motivic homotopy theory

Full Text: DOI

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