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Optimal embeddings into Lorentz spaces for some vector differential operators via Gagliardo's lemma. (English) Zbl 1442.46027

Atti Accad. Naz. Lincei, Cl. Sci. Fis. Mat. Nat., IX. Ser., Rend. Lincei, Mat. Appl. 30, No. 3, 413-436 (2019).

Summary: We prove a family of Sobolev inequalities of the form

$$\|u\|_{L^{\frac{n}{n-1},1}(\mathbb{R}^n,V)} \leq C\|A(D)u\|_{L^1(\mathbb{R}^n,E)}$$

where $A(D) : C_c^\infty(\mathbb{R}^n, V) \to C_c^\infty(\mathbb{R}^n, E)$ is a vector first-order homogeneous linear differential operator with constant coefficients, u is a vector field on $\mathbb{R}^n$ and $L^{\frac{n}{n-1},1}(\mathbb{R}^n)$ is a Lorentz space. These new inequalities imply in particular the extension of the classical Gagliardo-Nirenberg inequality to Lorentz spaces originally due to *A. Alvino* [Boll. Unione Mat. Ital., V. Ser., A 14, 148–156 (1977; Zbl 0352.46020)] and a sharpening of an inequality in terms of the deformation operator by *M. J. Strauss* [in: Partial diff. Equ., Berkeley 1971, Proc. Sympos. Pure Math. 23, 207–214 (1973; Zbl 0259.35008)] (Korn-Sobolev inequality) on the Lorentz scale. The proof relies on a nonorthogonal application of the Loomis-Whitney inequality and Gagliardo's lemma.

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MSC:

46E35	Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems	Cited in **7** Documents
26D10	Inequalities involving derivatives and differential and integral operators	
35A23	Inequalities applied to PDEs involving derivatives, differential and integral operators, or integrals	

Keywords:

Korn-Sobolev inequality; Lorentz spaces; Loomis-Whitney inequality

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