This article studies certain Banach Lie groups and homogeneous spaces for them that are attached to a unital $C^*$-algebra. The main object of interest is the tangent bundle $TG^+$ of the Banach Lie group of positive invertible elements. It carries a canonical action of the group of invertible elements. Even more, it is a homogeneous space for the unitary group of the indefinite Hermitian form $\langle (a_1, a_2), (b_1, b_2) \rangle = a_1^*b_1 - a_2^*b_2$. By a coordinate change, this Hermitian form is equivalent to $\langle (a_1, a_2), (b_1, b_2) \rangle = -i(a_1^*b_2 - a_2^*b_1)$. This coordinate change is analogous to the equivalence between the upper half plane and the open unit disk in the complex numbers. The paper also studies other approaches to this space and uses the different approaches to prove various properties of the space $TG^+$. One is concerned with different inner products on the same Hilbert space.

The $C^*$-algebra norm gives Finsler metrics on the various groups and homogeneous spaces studied in this paper. The article studies geodesics and covariant derivatives in this geometry. It is shown that $TG^+$ with this geometry is a length space with non-positive curvature.

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MSC:

46L05 General theory of $C^*$-algebras
58B20 Riemannian, Finsler and other geometric structures on infinite-dimensional manifolds
53C30 Differential geometry of homogeneous manifolds
22E65 Infinite-dimensional Lie groups and their Lie algebras: general properties
46L08 $C^*$-modules

Keywords:

homogeneous space; Banach Lie group; non-positive curvature; $C^*$-algebra

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