Hu, Yueke; Nelson, Paul D.; Saha, Abhishek
Some analytic aspects of automorphic forms on GL(2) of minimal type. (English)
Zbl 1443.11062

Summary: Let $\pi$ be a cuspidal automorphic representation of $\text{PGL}_2(\mathbb{A}_\mathbb{Q})$ of arithmetic conductor $C$ and Archimedean parameter $T$, and let $\phi$ be an $L^2$-normalized automorphic form in the space of $\pi$. The sup-norm problem asks for bounds on $\|\phi\|_\infty$ in terms of $C$ and $T$. The quantum unique ergodicity (QUE) problem concerns the limiting behavior of the $L^2$-mass $|\phi|^2(g)\,dg$ of $\phi$. Previous work on these problems in the conductor-aspect has focused on the case that $\phi$ is a newform. In this work, we study these problems for a class of automorphic forms that are not newforms. Precisely, we assume that for each prime divisor $p$ of $C$, the local component $\pi_p$ is supercuspidal (and satisfies some additional technical hypotheses), and consider automorphic forms $\phi$ for which the local components $\phi_p \in \pi_p$ are “minimal” vectors. Such vectors may be understood as non-archimedean analogues of lowest weight vectors in holomorphic discrete series representations of $\text{PGL}_2(\mathbb{R})$.

For automorphic forms as above, we prove a sup-norm bound that is sharper than what is known in the newform case. In particular, if $\pi_\infty$ is a holomorphic discrete series of lowest weight $k$, we obtain the optimal bound $C^{1/8-\varepsilon}k^{1/4-\varepsilon} \ll \varepsilon |\phi|_\infty \ll \varepsilon C^{1/8+\varepsilon}k^{1/4+\varepsilon}$. We prove also that these forms give analytic test vectors for the QUE period, thereby demonstrating the equivalence between the strong QUE and the subconvexity problems for this class of vectors. This finding contrasts the known failure of this equivalence [P. D. Nelson et al., J. Am. Math. Soc. 27, No. 1, 147–191 (2014; Zbl 1322.11051)] for newforms of powerful level.

MSC:
11F41 Automorphic forms on GL(2); Hilbert and Hilbert-Siegel modular groups and their modular and automorphic forms; Hilbert modular surfaces
11F11 Holomorphic modular forms of integral weight
11F30 Fourier coefficients of automorphic forms
11F70 Representation-theoretic methods; automorphic representations over local and global fields
11F85 $p$-adic theory, local fields
22E50 Representations of Lie and linear algebraic groups over local fields

Keywords:
automorphic form; automorphic representation; level aspect; minimal vectors; quantum unique ergodicity (QUE); sup-norm

Full Text: DOI

References:


S. Marshall, Local bounds for $L$-norms of Maass forms in the level aspect, Algebra Number Theory, 10 (2016), no. 4, 803-812. Zbl 1382.35175 MR 3519096 - Zbl 1382.35175


A. Saha, Hybrid sup-norm bounds for Maass newforms of powerful level, Algebra and Number Theory, 11 (2017), no. 5, 1009-1045. Zbl 06748165 MR 3671430 - Zbl 1432.11044


[45] H.

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.