

**Lan, Guitang; Sheng, Mao; Zuo, Kang**

**Semistable Higgs bundles, periodic Higgs bundles and representations of algebraic fundamental groups.** (English) [[Zbl 1444.14048](#)]

J. Eur. Math. Soc. (JEMS) 21, No. 10, 3053-3112 (2019).

The article under review establishes a  $p$ -adic version of Simpson's "Higgs versus Betti/de Rham" equivalence.

A "topological", or "Betti/de Rham", gadget here means a (relative) Fontaine module on a proper smooth  $X$  over Witt vectors of Hodge level  $\leq p - 2$ . The Higgs side is furnished by a new concept called "periodic Higgs-de Rham flow". To define this notion, a lift, mod  $p^n$ , of the Cartier transform of  $A$ . *Ogus* and *V. Vologodsky* [Publ. Math., Inst. Hautes Étud. Sci. 106, 1–138 (2007; [Zbl 1140.14007](#))] is constructed. The term "periodic" essentially accounts for the Frobenius action of the corresponding Fontaine module.

The authors also discuss the role of stability in this picture. Semistability of a graded Higgs bundle on the special fiber is shown to be equivalent to that the Higgs bundle supports a periodic Higgs-de Rham flow. The mod  $p$  stability of a 1-periodic Higgs object corresponds to the absolute irreducibility of the mod  $p$  representation of a crystalline  $\mathbf{Z}_p$ -representation under the equivalence.

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**MSC:**

- [14F30](#)  $p$ -adic cohomology, crystalline cohomology
- [14G20](#) Local ground fields in algebraic geometry
- [14H60](#) Vector bundles on curves and their moduli
- [14F40](#) de Rham cohomology and algebraic geometry
- [14F35](#) Homotopy theory and fundamental groups in algebraic geometry

Cited in **3** Reviews  
Cited in **7** Documents

**Keywords:**

[Higgs bundles](#); [p-adic Hodge theory](#)

**Full Text:** [DOI](#) [arXiv](#)

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