Bakhtawar, Ayreena; Bos, Philip; Hussain, Mumtaz
Hausdorff dimension of an exceptional set in the theory of continued fractions. (English)
Zbl 1446.11146
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The present article deals with continued fraction expansions of real numbers. That is, $[0,1) \ni x = [0 ; a_1(x), a_2(x), \ldots , a_n(x), \ldots]$, $a_n \in \mathbb{N}$.

The main attention is given to the Hausdorff dimension of the following set
$$\mathcal{F}(\Phi) = \left\{ x \in [0,1) : \begin{array}{ll} a_{n+1}(x)a_n(x) \geq \Phi(n) & \text{for infinitely many } n \in \mathbb{N} \text{ and } \\ a_{n+1}(x) < \Phi(n) & \text{for all sufficiently large } n \in \mathbb{N} \end{array} \right\},$$
where $\Phi : \mathbb{N} \to (1, \infty)$ is any function with $\lim_{n \to \infty} \Phi(n) = \infty$. In addition, the Hausdorff dimension of a certain generalization of the last-mentioned set, is considered.

The authors note that “the set $\mathcal{F}(\Phi)$ arises by excluding the set of well approximable points from the set of Dirichlet nonimprovable points expressed in terms of their continued fraction entries”.

Auxiliary results, notions, and surveys which are useful for obtaining the main results, are given. The importance of presented investigations is explained.

Reviewer: Symon Serbenyuk (Kyiv)

MSC:
11K50 Metric theory of continued fractions
11J70 Continued fractions and generalizations
11J83 Metric theory
28A78 Hausdorff and packing measures
28A80 Fractals

Keywords:
uniform Diophantine approximation; Hausdorff dimension; Dirichlet’s theorem; metric theory of continued fractions

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References:
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