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On gonality, scrolls, and canonical models of non-Gorenstein curves. (English) Zbl 1446.14016
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Let C be a curve (i.e. an integral and complete one-dimensional scheme over an algebraically closed field) of (arithmetic) genus g and let $C' \subseteq \mathbb{P}^{g-1}$ be its canonical model. In this paper the authors study the relation between the gonality of C and the dimension of a rational normal scroll S where C' can lie on, in particular when C is singular, or even non-Gorenstein, in which case $C' \not\cong C$. First, they analyze how to get an inclusion $C' \subset S$ from any pencil on C , in particular they get that S is $(d-1)$ -dimensional if C is d -gonal, thus extending to any gonality results by *R. Rosa* and *K.-O. Stöhr* [*J. Pure Appl. Algebra* 174, No. 2, 187–205 (2002; [Zbl 1059.14038](#))]. They also give an upper bound for the dimension of the singular set of S in terms of some invariants of the pencil, and look for sufficient conditions for S to be in fact singular. Then, in an opposite direction, they assume that C' lies on a given scroll S with prescribed dimension d and intersection number l with a generic fiber of S ; varying l , they are able to relate properties of C , such as gonality and the kind of its singularities, with d and other invariants of S . This leads to a generalization to arbitrary d of some results by *D. Lara* et al. [*Int. J. Math.* 27, No. 5, Article ID 1650045, 30 p. (2016; [Zbl 1357.14040](#))]. At the end, they consider rational monomial curves and prove that such curves have gonality d if and only if their canonical model lies on a $(d-1)$ -fold scroll, and does not lie on any scroll of smaller dimension.

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MSC:

- [14H20](#) Singularities of curves, local rings
- [14H45](#) Special algebraic curves and curves of low genus
- [14H51](#) Special divisors on curves (gonality, Brill-Noether theory)

Keywords:

[non-Gorenstein curve](#); [canonical model](#); [gonality](#); [scrolls](#)

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