

Vidal Martins, Renato; Lara, Danielle; Menezes Souza, Jairo

On gonality, scrolls, and canonical models of non-Gorenstein curves. (English) Zbl 1446.14016
Geom. Dedicata 203, 111-133 (2019).

Let C be a curve (i.e. an integral and complete one-dimensional scheme over an algebraically closed field) of (arithmetic) genus g and let $C' \subseteq \mathbb{P}^{g-1}$ be its canonical model. In this paper the authors study the relation between the gonality of C and the dimension of a rational normal scroll S where C' can lie on, in particular when C is singular, or even non-Gorenstein, in which case $C' \not\cong C$. First, they analyze how to get an inclusion $C' \subset S$ from any pencil on C , in particular they get that S is $(d-1)$ -dimensional if C is d -gonal, thus extending to any gonality results by *R. Rosa* and *K.-O. Stöhr* [*J. Pure Appl. Algebra* 174, No. 2, 187–205 (2002; [Zbl 1059.14038](#))]. They also give an upper bound for the dimension of the singular set of S in terms of some invariants of the pencil, and look for sufficient conditions for S to be in fact singular. Then, in an opposite direction, they assume that C' lies on a given scroll S with prescribed dimension d and intersection number l with a generic fiber of S ; varying l , they are able to relate properties of C , such as gonality and the kind of its singularities, with d and other invariants of S . This leads to a generalization to arbitrary d of some results by *D. Lara et al.* [*Int. J. Math.* 27, No. 5, Article ID 1650045, 30 p. (2016; [Zbl 1357.14040](#))]. At the end, they consider rational monomial curves and prove that such curves have gonality d if and only if their canonical model lies on a $(d-1)$ -fold scroll, and does not lie on any scroll of smaller dimension.

Reviewer: [Caterina Cumino \(Torino\)](#)

MSC:

- [14H20](#) Singularities of curves, local rings
- [14H45](#) Special algebraic curves and curves of low genus
- [14H51](#) Special divisors on curves (gonality, Brill-Noether theory)

Keywords:

non-Gorenstein curve; canonical model; gonality; scrolls

Full Text: [DOI](#) [arXiv](#)

References:

- [1] Andreotti, A., Mayer, A.L.: On period relations for abelian integrals on algebraic curves. *Annali della Scuola Normale Superiore di Pisa* 21(2), 189-238 (1967) · [Zbl 0222.14024](#)
- [2] Babbage, D.W.: A note on the quadrics through a canonical curve. *J. Lond. Math. Soc.* 14, 310-315 (1939) · [Zbl 65.1398.03](#) · [doi:10.1112/jlms/s1-14.4.310](#)
- [3] Bresinsky, H.: Monomial space curves in \mathbb{A}^3 as set-theoretic complete intersections. *Proc. Am. Math. Soc.* 75, 23-24 (1979) · [Zbl 0395.14015](#)
- [4] Barucci, V., Fröberg, R.: One-dimensional almost Gorenstein rings. *J. Algebra* 188, 418-442 (1997) · [Zbl 0874.13018](#) · [doi:10.1006/jabr.1996.6837](#)
- [5] Brundu, M., Sacchiero, G.: Stratification of the moduli space of four-gonal curves. *Proc. Edinb. Math. Soc.* 57(03), 631-686 (2014) · [Zbl 1304.14031](#) · [doi:10.1017/S001309151300062X](#)
- [6] Casnati, G., Ekedahl, T.: Covers of algebraic varieties. I. A general structure theorem, covers of degree 3, 4 and Enriques surfaces. *J. Algebraic Geom.* 5(3), 439-460 (1996) · [Zbl 0866.14009](#)
- [7] Contiero, A., Stöhr, K.-O.: Upper bounds for the dimension of moduli spaces of curves with symmetric Weierstrass semigroups. *J. Lond. Math. Soc.* 88, 580-598 (2013) · [Zbl 1288.14016](#) · [doi:10.1112/jlms/jdt034](#)
- [8] Contiero, A., Feital, L., Martins, R.V.: Max Noether theorem for integral curves. *J. Algebra* 494, 111-136 (2018) · [Zbl 1386.14112](#) · [doi:10.1016/j.jalgebra.2017.10.009](#)
- [9] Coppens, M.: Free linear systems on integral Gorenstein curves. *J. Algebra* 145, 209-218 (1992) · [Zbl 0770.14002](#) · [doi:10.1016/0021-8693\(92\)90186-P](#)
- [10] Cotterill, E., Feital, L., Martins, R. V.: Dimension counts for singular rational curves via semigroups. [arXiv:1511.08515v2](#) · [Zbl 1394.14019](#)

- [11] Cotterill, E., Feital, L., Martins, R.V.: Singular rational curves with points of nearly-maximal weight. *J. Pure Appl. Algebra* 222, 3448-3469 (2018). <https://doi.org/10.1016/j.jpaa.2017.12.017> · [Zbl 1394.14019](#) · [doi:10.1016/j.jpaa.2017.12.017](#)
- [12] Eisenbud, D., Harris, J., Koh, J., Stillmann, M.: Determinantal equations for curves of high degree. *Am. J. Math.* 110, 513-539 (1988) · [Zbl 0681.14027](#) · [doi:10.2307/2374621](#)
- [13] Eisenbud, D., Harris, J.: On varieties of minimal degree. *Proc. Symp. Pure Math.* 46, 3-13 (1987) · [doi:10.1090/pspum/046.1/927946](#)
- [14] Enriques, F.: Sulle curve canoniche di genera $\lfloor p \rfloor$ nello spazio a $\lfloor p-1 \rfloor$ dimensioni. *Rend. Accad. Sci. Ist. Bologna* 23, 80-82 (1919)
- [15] Herzog, J.: Generators and relations of abelian semigroups and semigroup rings. *Manuscr. Math.* 3, 175-193 (1970) · [Zbl 0211.33801](#) · [doi:10.1007/BF01273309](#)
- [16] Hotchkiss, J., Ullery, B.: The gonality of complete intersection curves. [arXiv:1706.08169](#)
- [17] Jäger, J.: Längeberechnungen und Kanonische Ideale in Eindimensionalen Ringen. *Arch. Math.* 29, 504-512 (1977) · [Zbl 0374.13006](#) · [doi:10.1007/BF01220445](#)
- [18] Kleiman, S.L., Martins, R.V.: The canonical model of a singular curve. *Geom. Dedicata* 139, 139-166 (2009) · [Zbl 1172.14019](#) · [doi:10.1007/s10711-008-9331-4](#)
- [19] Lara, D., Marchesi, S., Martins, R.V.: Curves with canonical models on scrolls. *Int. J. Math.* 27(5), 1650045-1-30 (2016) · [Zbl 1357.14040](#) · [doi:10.1142/S0129167X16500452](#)
- [20] Martins, R.V.: On trigonal non-Gorenstein curves with zero Maroni invariant. *J. Algebra* 275, 453-470 (2004) · [Zbl 1060.14036](#) · [doi:10.1016/j.jalgebra.2003.10.033](#)
- [21] Matsuoka, T.: On the degree of singularity of one-dimensional analytically irreducible noetherian rings. *J. Math. Kyoto Univ.* 11, 485-491 (1971) · [Zbl 0224.13017](#) · [doi:10.1215/kjm/1250523616](#)
- [22] Miró-Roig, R.M.: The representation type of rational normal scrolls. *Rend. Circ. Mat. Palermo* 62, 153-164 (2012) · [Zbl 1268.14014](#) · [doi:10.1007/s12215-013-0113-y](#)
- [23] Reid, M.: Chapters on algebraic surfaces. 6 Feb 1996. Lectures of a summer programm Park City, UT. [arXiv:alg-geom/9602006v1](#) (1993)
- [24] Rosa, R., Stöhr, K.-O.: Trigonal Gorenstein curves. *J. Pure Appl. Algebra* 174, 187-205 (2002) · [Zbl 1059.14038](#) · [doi:10.1016/S0022-4049\(02\)00122-6](#)
- [25] Rosenlicht, M.: Equivalence relations on algebraic curves. *Ann. Math.* 56, 169-191 (1952) · [Zbl 0047.14503](#) · [doi:10.2307/1969773](#)
- [26] Schreyer, F.-O.: Syzygies of canonical curves and special linear series. *Mathematische Annalen* 275, 105-137 (1986) · [Zbl 0578.14002](#) · [doi:10.1007/BF01458587](#)
- [27] Stöhr, K.-O.: On the poles of regular differentials of singular curves. *Boletim da Sociedade Brasileira de Matemática* 24, 105-135 (1993) · [Zbl 0788.14020](#) · [doi:10.1007/BF01231698](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.